Dynamic Aperture Control for Synchrotrons

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Outline

1.0 Framework: $\dot{\vec{p}} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Lie Series

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Truncated Power Series

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Symplectic Integrator

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Map Normal Form

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Radiation

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Polymorphism

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Accelerator Design

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Control Theory

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2.0 Overview

2.1 Lattice Design for Synchrotron Light Sources

Challenge (D. H. Bilderbak, P. Elleaume, and Edgar Weckert, 2005):

"*One of the most important design challenges of such a lattice involves the enlargement of the dynamic aperture in order to reach a value sufficient for injection*."

Given a Hamiltonian system

$$\boxed{H = H_0 + \varepsilon V}$$

Approach: Analyze the properties of the corresponding Poincaré map.

Lie series approach (Dragt and Finn, 1976):

$$\mathcal{M}_{0\to1} = e^{:h_1:}e^{:h_2:}e^{:h_3:}e^{:h_4:}\ldots, \qquad :f:g \equiv [f, g] \equiv \sum_{i=1}^{2n}\left[\frac{\partial f}{\partial x_i}\frac{\partial g}{\partial p_{xi}} - \frac{\partial g}{\partial x_i}\frac{\partial f}{\partial p_{xi}}\right]$$

"It also provides a new approach since the connection between symplectic maps, Lie algebras, invariant functions, and Birkhoff's work has not been previously recognized and exploited. It is expected that the results obtained will be applicable to the normal form problem in Hamiltonian mechanics, the use of the Poincaré section map in stability analysis, and the behavior of magnetic field lines in a toroidal plasma device."

Hamiltonian approach (A. Bazzani, P. Mazzanti, G. Servizi, and G. Turchetti, 1988):

"We describe the motion of a particle in the lattice of a hadron accelerator using the formalism of symplectic maps. We revisit the Courant-Snyder's theory and we stress that the reduction to normal form of a symplectic map is just the natural generalization of the linear theory."

A recursive (arbitrary order) map normal form algorithm (Forest, 1990):

$$\mathcal{M} \cong \mathcal{A}^{-1} e^{:-g\left(\vec{J}, \vec{\phi}\right):} e^{:k\left(\vec{J}\right):} \mathcal{R}\, e^{:g\left(\vec{J}, \vec{\phi}\right):} \mathcal{A},$$

$$\nu_{x,y} = -\frac{1}{2\pi}\frac{\partial k\left(\vec{J}\right)}{\partial J_{x,y}}, \qquad \beta_{xi} + \Delta\beta_{xi} = \langle e^{:-g\left(\vec{J}, \vec{\phi}\right):} \mathcal{R}_{n \to i} \mathcal{A}_i x_i^2 \rangle_{\vec{\phi}}, \qquad \text{etc.}$$

Problem (Poincaré, 1892): perturbation theory is limited by the “small denominator problem” from celestial mechanics.

Theorem: Kolmogorov-Arnold-Moser (1954-1963). Roughly, a system with periodic solution has quasi-periodic solutions for sufficiently small ε.

Problem (Percival, 1986):

“*In fact, Hénon showed Arnold’s proof only applies if the perturbation is less than 10^{-333} and Moser’s if it is less than 10^{-48}, in appropriate units. The latter is less than the gravitation perturbation of a football in Spain by the motion of a bacterium in Australia!*”

Theorem: Nekhoroshev. Roughly, the confinement time scales exponentially as ε tends to zero.

Problem (Warnock and Ruth, 1991). "*Unfortunately, the Nekhoroshev Theorem has no direct practical application, since ε must be absurdly small to guarantee a stability time T of suitable magnitude. This situation results from pessimistic estimates that are required in the rigorous analysis.*"

Approach: analyze the Lie generator (Bengtsson and Irwin, SSC 1990):

$$\boxed{\begin{aligned}\mathcal{M} &= \mathcal{M}^{\text{linear}}_{1\to 2} e^{:g_{3,2}:} \mathcal{M}^{\text{linear}}_{2\to 3} e^{:g_{3,3}:} \ldots \mathcal{M}^{\text{linear}}_{n-2\to n-1} e^{:g_{3,n-1}:} \mathcal{M}^{\text{linear}}_{n-1\to n} \\ &= e^{:\hat{g}_{3,2}:} e^{:\hat{g}_{3,4}:} \ldots e^{:\hat{g}_{3,n-1}:} \mathcal{M}^{\text{linear}}_{1\to n} = \mathcal{A}^{-1} e^{:h_3+h_4+h_5+\ldots:} \mathcal{R}\mathcal{A}\end{aligned}}$$

Problem: Taylor series have a finite radius of convergence, and we still lack a theorem for stability in the general non-linear case.

Intuitive approach: reduce the magnitude of the Lie generator and evaluate by numerical simulations of the system of ODEs (tracking).

Challenge: how to build a self-consistent model that includes the effects of engineering tolerances and radiation?

Solution exists: Forest, Hirata, Chao, et al.

Problem: how to (correctly) implement the entire framework in an effective manner.

Approach (Bengtsson and Forest, 1996): integrate the algorithms for symplectic integrator-, Truncated Power Series Algebra-, and Map Normal Form with modern computer programming techniques (a polymorphic beamline class: Thor). In particular, a successful object-oriented implementation must be guided by the mathematical structures, not vice-verse.

2.2 Accelerator Control (Single Particle Dynamics)

Challenge (R. Bartolini and F. Schmidt, 1998):

"*Since many years perturbation theory [1] and more recently the Normal Form [2, 3] techniques have been used to understand nonlinear motion of single particles in hadron accelerators. This has proven to be very useful in the design phase of an accelerator. When it comes to existing machines these sophisticated tools have been rarely in use up to now. In part this is due to the complexity of the theory but also due to the fact that a nonlinear model of the accelerator cannot be easily anticipated. Checking such a model experimentally [4] may prove even more difficult.*

One well documented attempt to overcome this problem has been made by Bengtsson [5]. In the framework of the first order perturbation theory he has studied how the real spectra from tracking or experimental turn-by-turn data can be related to resonances. This study has stopped short of a complete solution. An important prerequisite to his analysis was a tune measurement technique superior to the standard FFT [6]. Similar attempts were performed in the field of celestial mechanics [7]."

Complimentary approaches:

- **Frequency Map Analysis (D. Robin, C. Steier, L. Nadolski, and Laskar, 2000).**
- **Linear Optics from Closed Orbits (J. Safranek, 1996).**

3.0 Control Theory

Challenge: Given a model for a dynamical system, how to improve the performance?

3.1 Classical Control Theory

A linear, time-invariant, single-input/single-output systems (Nyquist, 1932, Bode, 1940)

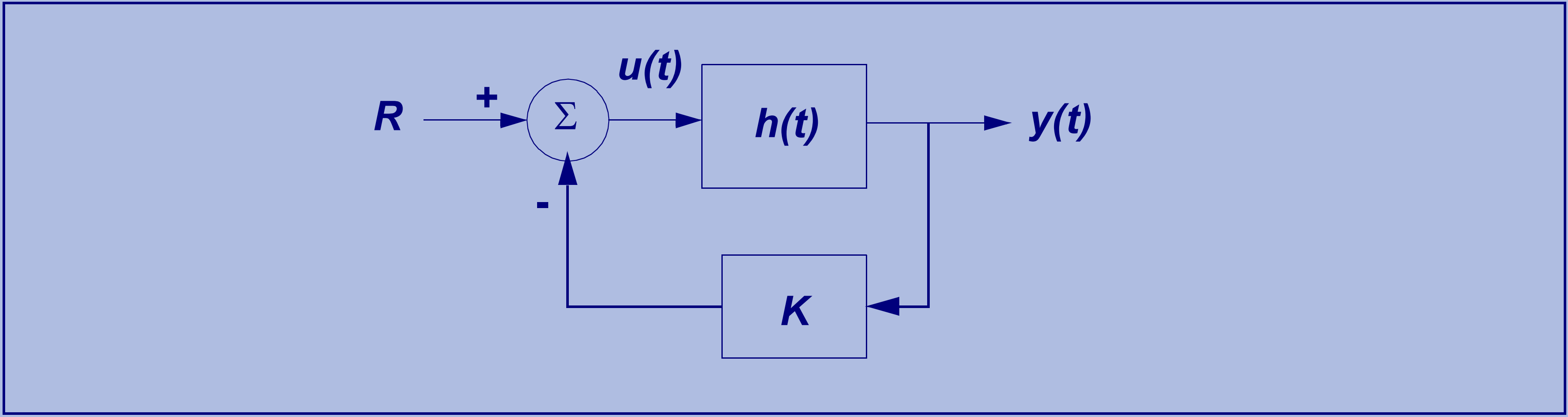

FIGURE 1. Transfer Function (transient behavior).

3.2 Modern Control Theory

A linear, time-varying system with multiple input/multiple outputs (R. Kalman, 1960)

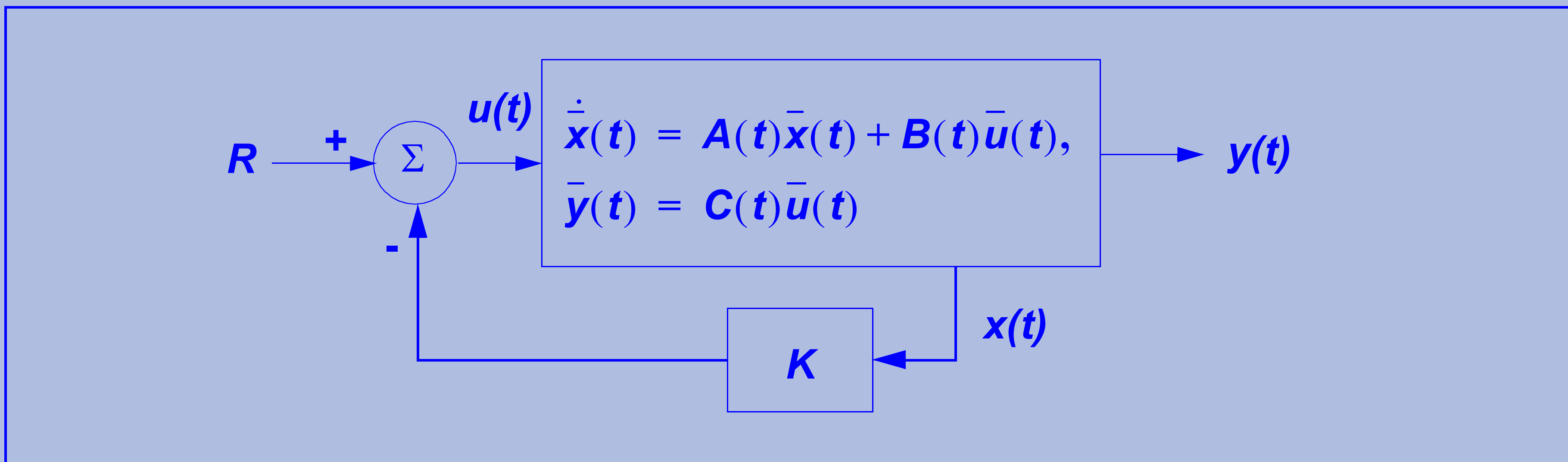

FIGURE 2. State-Space Approach (full-state feedback).

This concept is the theme for the remaining slides.

Similarly, the corresponding discrete-time system is

$$\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k,$$
$$\bar{y}_k = C\bar{u}_k$$

The dynamics of the system is determined by the eigenvalues of A

$$A^N = T\Gamma^N T^{-1}$$

Moreover, the system is controllable and observable if

$$W_c = \begin{bmatrix} B & AB \end{bmatrix}, \qquad W_o = \begin{bmatrix} C \\ CA \end{bmatrix}, \qquad \mathrm{rank}(W) = n$$

For the latter, all the internal states can be estimated from a single output signal

$$\bar{x}_k = AW_o^{-1}\begin{bmatrix} y_{k-1} \\ y_k \end{bmatrix} + \left\{ [BA^{-1}B] - AW_o^{-1}\begin{bmatrix} 0 & 0 \\ CB & 0 \end{bmatrix} \right\} \begin{bmatrix} u_{k-1} \\ u_k \end{bmatrix}$$

Summary:

The intrinsic properties of linear systems:

- **stability,**
- **controllability,**
- **and observability**

are determined by purely algebraic properties, i.e.:

- **the eigenvalues of the state matrix,**
- **and certain rank conditions of the state-space matrices.**

i.e. a coordinate-free description. In particular, stability is independent of the initial conditions. This is typically not the case for non-linear systems. -> Numerical simulations (tracking).

3.3 Particle Accelerator (Linear) Control Theory

Hill's equation

$$x'' + K(s)x = 0$$

can be written

$$p_x' = -K(s)x, \qquad x' = p_x$$

with the discrete-time version (Courant and Snyder, 1958)

$$\bar{x}_{k+1} = M\bar{x}_k, \qquad \bar{x}_k \equiv [x_k, p_{xk}]^{\mathrm{T}}$$

The state (transport) matrix

$$\vec{x}_1 = M_{0\to1}\vec{x}_0, \qquad \vec{x} = [x, p_x, y, p_y, c_0 t, \delta]^{\mathrm{T}}$$

is concatenated by matrix multiplication (Lorentz invariant, symplectic group.)

$$M_{0\to2} = M_{1\to2}M_{0\to1}$$

Generalizing, the driven pseudo-harmonic oscillator

$$\boxed{x'' + K(s)x = u(s)}$$

leads to

$$\boxed{\begin{aligned} \bar{x}_{k+1} &= M\bar{x}_k + B\bar{u}_k, \\ \bar{y}_k &= C\bar{u}_k \end{aligned}}$$

It is straightforward to show that the system is both controllable and observable. Correspondingly, one can:

- Monitor the (linear) phase-space motion with one BPM.
- Design and implement feed-back systems for e.g. coherent bunch instabilities.

4.0 A “Wind Tunnel” for the System of ODEs

Challenge: How to implement one model for both analytical- and numerical studies, with a self-consistent treatment of the impact of engineering tolerances and radiation?

4.1 Equations of Motion

Note, expansions in “time”, i.e. the *s*-coordinate.

Hamiltonian (phase-space: $\vec{x} = [x, p_x, y, p_y, \delta, c_0 \Delta t]$)

$$H = -(1 + h_{\mathrm{ref}}(s)x)\left[\frac{q}{p_0}A_s(s) + \sqrt{(1+\delta)^2 - \left(p_x - \frac{q}{p_0}A_x(s)\right)^2 - \left(p_y - \frac{q}{p_0}A_y(s)\right)^2}\right]$$

A 4th-order symplectic integrator is given by (Forest and Ruth, Yoshida, 1990)

$$\mathcal{S}_4(L) \equiv e^{:-c_1 L H_d:} e^{:-d_1 L H_k:} e^{:-c_2 L H_d:} e^{:-d_2 L H_k:} e^{:-c_2 L H_d:} e^{:-d_1 L H_k:} e^{:-c_1 L H_d:} + O(L^5),$$

$$c_1 = \frac{1}{2(2-2^{1/3})}, \qquad c_2 = \frac{1-2^{1/3}}{2(2-2^{1/3})}, \qquad d_1 = \frac{1}{2-2^{1/3}}, \qquad d_2 = -\frac{2^{1/3}}{2-2^{1/3}}$$

Problem: how to model mechanical mis-alignments?

Solution: introduce an Euclidian transformation before- and after each element.

Problem: how to introduce classical radiation?

Solution: generalize from Poisson bracket $:H_k:$ to vector field $V_k \cdot \nabla$.

Problem: How to extract the corresponding Taylor maps to arbitrary order?

Solution: Replace the numerical operations (+,-,*,/) on the phase space vector with TPSA.

Problem: How to obtain the equilibrium emittance?

Solution: Compute the (complex) eigenvalues of the linear one-turn map and the diffusion coefficients related to $[\langle x^2\rangle, \langle y^2\rangle, \langle z^2\rangle]$ for the eigenvectors.

4.2 Truncated Power Series Algebra (TPSA)

Truncated power series

$$P = \sum_{|\vec{i}|=0}^{N} a_{\vec{i}} x^{\vec{i}}, \qquad \vec{i} \equiv (i_1, i_2, \ldots, i_n), \qquad |\vec{i}| \equiv i_1 + i_2 + \ldots + i_n$$

$$x \equiv (x_1, x_2, \ldots, x_n), \qquad x^{\vec{i}} \equiv x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n}$$

form an (associative) algebra over the real numbers

$$(P \cdot Q) \cdot R = P \cdot (Q \cdot R) + O(x^{N+1}), \qquad (P + Q) \cdot R = P \cdot R + Q \cdot R + O(x^{N+1}),$$

$$P \cdot (Q + R) = P \cdot Q + P \cdot R + O(x^{N+1}),$$

$$a \cdot (P \cdot Q) = (a \cdot P) \cdot Q + O(x^{N+1}) = P \cdot (a \cdot Q) + O(x^{N+1})$$

Moreover (inverse)

$$\frac{1}{I + P} = I - P + P^2 + \ldots \pm P^N + O(x^{N+1})$$

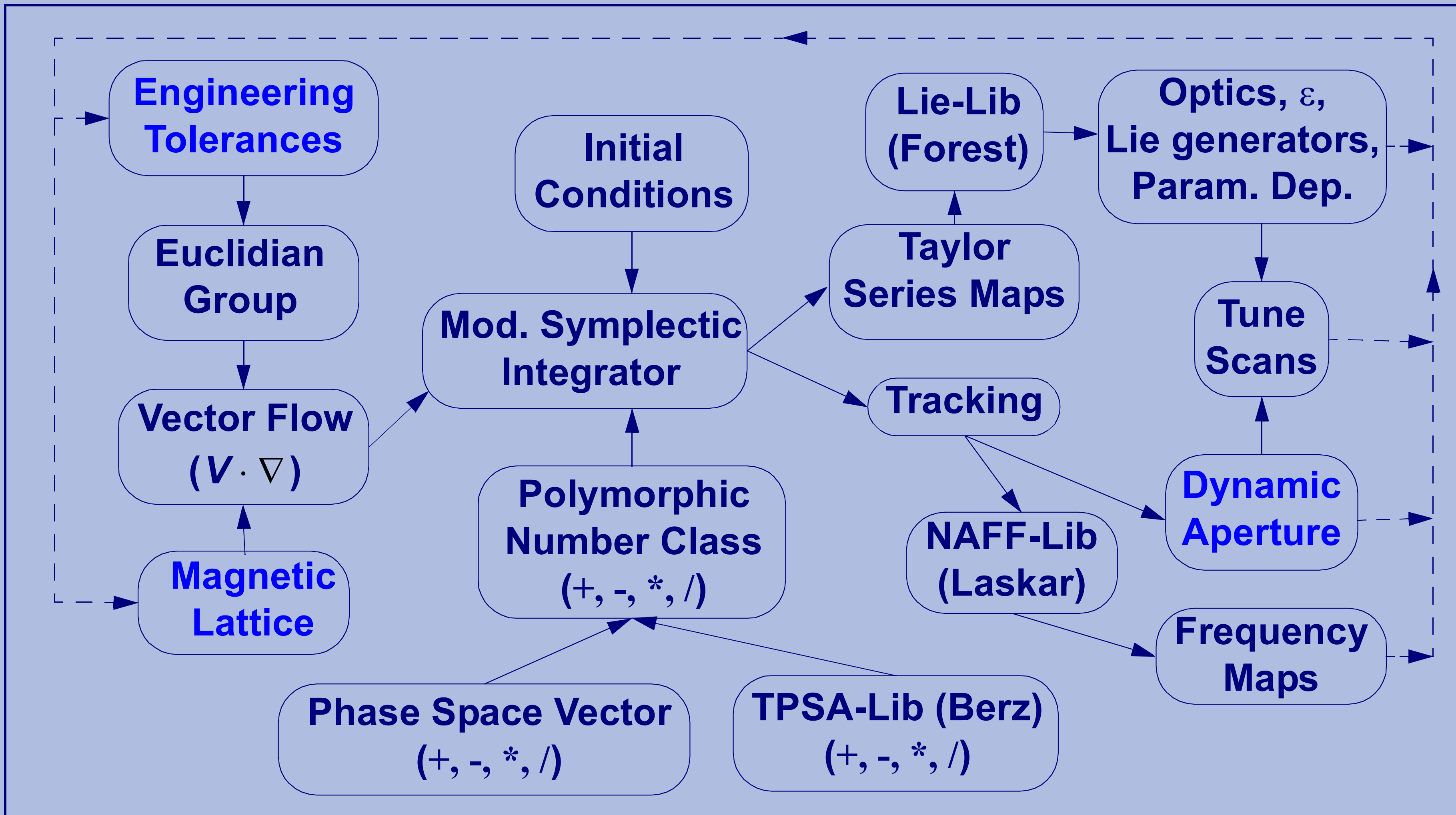

FIGURE 3. The Polymorphic Tracking Code: Information Flow (~50,000 lines of C++, C, and FORTRAN code).

```
/* drift */
template<typename T>
void drift_pass(const T L, ss_vect<T> &x)
{
  T  ps, u;

  if (!nl_drift_on) {
    u = L/(1.0+x[delta_]);
   x[ct_] += u*(pow(x[px_], 2)+pow(x[py_], 2))/(2.0*(1.0+x[delta_]));
  } else {
    ps = get_ps(x); u = L/ps; x[ct_] += u*(1.0+x[delta_]) - L;
  }
  x[x_] += x[px_]*u; x[y_] += x[py_]*u;
  if (totpath_on) x[ct_] += L;
}
```

FIGURE 4. Polymorphic Propagator for a Drift (C++).

To summarize, an integrated implementation of the algorithms for:

- symplectic integrator,
- TPSA,
- and the Map Normal Form

by polymorphism (operator overloading) provides a generic tool to compute:

- any global property,
- to arbitrary order,
- with self-consistent modeling of engineering tolerances and radiation;
- and parameter dependence.

In particular, all quantities are computed from the same equations of motion (Hamiltonian, vector flow) and algorithm for numerical integration.

Problem: since the Map Normal Form approach is fundamentally no different from classical perturbation theory, the computed global properties and invariants are limited by the “small denominator problem” from celestial mechanics.

Challenge: Lacking a control theory for the general non-linear case, what to do?

5.0 Dynamic Aperture: a Control Theory Approach

Note, expansions in the multipole strengths.

Challenge: How to control the (linear) chromaticity while maintaining adequate dynamic aperture?

5.1 Lie Generator Approach

The one-turn map

$$\vec{x}_{n+1} = \mathcal{M}\vec{x}_n$$

leads to the stability problem

$$\vec{x}_N = \mathcal{M}^N\vec{x}_0$$

The map can be written

$$\begin{aligned}\mathcal{M} &= \mathcal{M}^{\text{linear}}_{1\to 2}e^{:g_{3,2}:}\mathcal{M}^{\text{linear}}_{2\to 3}e^{:g_{3,3}:}\ldots\mathcal{M}^{\text{linear}}_{n-2\to n-1}e^{:g_{3,n-1}:}\mathcal{M}^{\text{linear}}_{n-1\to n}\\ &= e^{:\hat{g}_{3,2}:}e^{:\hat{g}_{3,4}:}\ldots e^{:\hat{g}_{3,n-1}:}\mathcal{M}^{\text{linear}}_{1\to n} = \mathcal{A}^{-1}e^{:h_3+h_4+h_5+\ldots:}\mathcal{R}\mathcal{A}\end{aligned}$$

by introducing similarity transformations

$$\mathcal{M}_{\mathrm{linear}} e^{:g:} \mathcal{M}_{\mathrm{linear}}^{-1} = e^{:\mathcal{M}_{\mathrm{linear}} g:} = e^{:\hat{g}:}$$

and the CBH formula

$$e^{:f:} e^{:g:} = e^{:f + g + [f, g]/2 + O(3):}$$

In other words:

1. Transform to Floquet space (normalized phase space).
2. Parallel transport all the (thin) nonlinear kicks to the beginning of the lattice.
3. Combine them into a single (thin) nonlinear kick with the CBH formula.

The stability problem then takes the form

$$\boxed{\mathcal{M}^{N} = \mathcal{A}^{-1}\left(e^{:h_3 + h_4 + \ldots:}\mathcal{R}\right)^{N}\mathcal{A}}$$

To summarize, the dynamic aperture problem has been factored into two parts:

- A non-linear Lie generator (thin kick) *h*,
- and a phase-space rotation $\mathcal{R}$ (working point).

Intuitive approach:

- Minimize *h* to bring the map closer to the linear approximation.
- Determine the dynamic aperture from tracking.
- Optimize the tune.
- Iterate => robust dynamic aperture and optics design becomes tightly coupled for high performance lattices.
- Include engineering tolerances and radiation.
- Refine.

The vector potential for a (thin) sextupole at location s_i is

$$\frac{q}{p_0}A_s(s_i) = -\frac{b_{3i}}{3}(x^3 - 3xy^2).$$

Introducing the resonance basis

$$h_x^{\pm} = \sqrt{2J_x}e^{\pm\phi_x} = \sqrt{2J_x}\cos(\phi_x) \pm \sqrt{2J_x}\sin(\phi_x) = x \mp ip_x$$

and the mode expansion

$$h_3 \equiv \sum_{|\bar{i}| = n} h_{\bar{i}} h_x^{+i_1} h_x^{-i_2} h_y^{+i_3} h_x^{-i_4} \delta^{i_5},$$

where (can be represented as phasor sums: $(\mathrm{Re}\{h_i\}, \mathrm{Im}\{h_i\})$)

$$h_{jklmp} = h_{jklmp}^{*} = \frac{1}{A}\sum_{n=1}^{N}(b_{3n}L)\beta_{xn}^{(j+k)/2}\beta_{yn}^{(l+m)/2}\eta_x^p e^{i[(j-k)\mu_{xi} + (l-m)\mu_{yi}]},$$

First order generators:

Phenomena	Generators
Linear Chromaticity	h_{11001}, h_{00111}
Sextupolar (geometric) Modes	h_{10110}, h_{21000}, h_{30000}, h_{10020}, h_{10200}
Chromatic (synchro-betatron) Modes	h_{20001}, h_{00201}
Second Order Dispersion	h_{10002}

Table 1: First Order Sextupolar Generators.

Turning the crank to second order

$$h_4 = \frac{1}{J_x^{\alpha} J_y^{\beta}} \sum_{|\vec{I}| = |\vec{J}| = n} h_{\vec{I}} h_{\vec{J}} h_x^{+(i_1+j_1)} h_x^{-(i_2+j_2)} h_y^{+(i_3+j_3)} h_x^{-(i_4+j_4)} \delta^{(i_5+j_5)},$$
$$\alpha + \beta = 2$$

where (phase independent)

$$\begin{aligned}(h_{\vec{I}} h_{\vec{J}})_{\mathrm{Ker}} = &-\frac{1}{64}(3h_{21000}h_{12000} + h_{30000}h_{03000})(2J_x)^2 \\ &+\frac{1}{16}(2h_{21000}h_{01110} + h_{10020}h_{01200} + h_{10200}h_{01020})(2J_x)(2J_y) \\ &-\frac{1}{64}(4h_{10110}h_{01110} + h_{10020}h_{01200} + h_{10200}h_{01020})(2J_y)^2\end{aligned}$$

and (drive sextupolar modes)

$$
\begin{aligned}
(h_{\bar{l}}h_{\bar{J}})_{\mathrm{Im}} = & -\frac{1}{64}[2(h_{30000}h_{12000})_{2\nu_x} + (h_{30000}h_{21000})_{4\nu_x}](2J_x)^2 \\
& + \frac{1}{64}[2(h_{30000}h_{01110} + h_{21000}h_{10110} + 2h_{10200}h_{10020})_{2\nu_x} \\
& + 2(h_{10200}h_{12000} + h_{21000}h_{01200} + 2h_{10200}h_{01110} + 2h_{10110}h_{01200})_{2\nu_y} \\
& + (h_{21000}h_{10020} + h_{30000}h_{01020} + 4h_{10110}h_{10020})_{2\nu_x - 2\nu_y} \\
& + (h_{30000}h_{01200} + h_{10200}h_{21000} + 4h_{10110}h_{10200})_{2\nu_x + 2\nu_y}](2J_x)(2J_y) \\
& + \frac{1}{64}[2(h_{10200}h_{01110} + h_{10110}h_{01200})_{2\nu_y} + (h_{10200}h_{01200})_{4\nu_y}](2J_y)^2 + \text{c.c.}
\end{aligned}
$$

Notes:

1. **Since the second order generators are cross terms of the first order, they can be controlled by local cancellation of the first order.**

2. **It is clear that $(h_{\bar{l}}h_{\bar{J}})_{\mathrm{Ker}}$ drive amplitude dependent tune shifts. However, $(h_{\bar{l}}h_{\bar{J}})_{\mathrm{Im}}$ contribute too, i.e. when the map is brought to normal form.**

Generalizing to 3rd order:

- 2nd order brings 8 more geometric modes.
- 3rd order adds another 14 geometric modes.
- And, chromatic (synchro-betatron) modes as well.
- 2nd-, 4th- and 6th order introduces amplitude dependent tune shifts (12), higher order chromaticity (4), and cross terms (7).

1st Order (5)	(1, 0)	(3, 0)	(1, -2)	(1, 2)					
2nd Order (8)	(2, 0)	(0, 2)	(4, 0)	(0, 4)	(2, -2)	(2, 2)			
3rd Order (14)	(1, 0)	(3, 0)	(1, -2)	(1, 2)	(5, 0)	(1, -4)	(1, 4)	(3, -2)	(3, 2)

Table 2: Sextupolar Geometric Modes: $(n_x \nu_x, n_y \nu_y)$.

$(h_I h_{\bar{J}})_{\mathrm{Ker}}$:

Order	Geometric Generators
2	$h_{22000}J_x^2,\ h_{11110}J_xJ_y,\ h_{00220}J_y^2$
4	$h_{33000}J_x^3,\ h_{22110}J_x^2J_y,\ h_{11220}J_xJ_y^2,\ h_{00330}J_y^3$
6	$h_{44000}J_x^4,\ h_{33110}J_x^3J_y,\ h_{22220}J_x^2J_y^2,\ h_{11330}J_xJ_y^3,\ h_{00440}J_y^3$

Table 3: Sextupolar Geometric Generators of Amplitude Dependent Tune Shifts.

Order	Chromatic Generators
1	$h_{11001}J_x\delta,\ h_{00111}J_y\delta,\ h_{11002}J_x\delta^2,\ h_{00112}J_y\delta^2,\ h_{11003}J_x\delta^3,\ h_{00113}J_y\delta^3$
2	$h_{22001}J_x^2\delta,\ h_{11111}J_xJ_y\delta,\ h_{00221}J_y^2\delta$
4	$h_{22111}J_x^3\delta,\ h_{22111}J_x^2J_y\delta,\ h_{11221}J_xJ_y^2\delta,\ h_{00331}J_y^3\delta$

Table 4: Sextupolar Chromatic Generators of Nonlinear Chromaticity.

5.2 Application to NSLS-II

Approach:

1. Extract the one-turn map (to 8th order in phase-space coordinates); with parameter dependence.
2. Compute the corresponding Lie generator (to 9th order, i.e. 3rd order in the sextupole strengths); with parameter dependence.
3. The non-linear part of the dynamic aperture problem has now been reduced to a 52×9 system of linear, quadratic, and cubic equations for the sextupole strengths.
4. Determine a merit function with suitable (heuristic) weights by testing the effect on the dynamic aperture from tracking.
5. Optimize the tune: select a grid of working points, adjust the cell tune, determine the sextupole strengths (by minimizing the 52×9 system), and evaluate the dynamic aperture from tracking.

This is essentially the strategy we developed for the Swiss Light Source conceptual design, and pursued analytically to second order (with computer algebra), by instrumenting the optics code with the corresponding analytical formula.

Active logbook

Machine Shift Summaries

Entry date	Priority	Author
22/02/2005 07:00	N	Andreas Luedeke

Title

22.Feb Früh, Lifetime studies and F4TJB test

Keywords

Machine Shift

Logbook entry

Machine development

(Andersson, Pedrozzi, Streun)

Summary (detailed evaluation will follow):

- Test of a new sextupole setting (optics F4TJB calculated by J.Bengtsson, BNL): better injection efficiency (100% without any optimization!) (due to larger horizontal DA) but lower lifetime (due to larger 2nd order chromaticity). Tests for various tunes and chroma - quite promising optics.
- Chromaticity measurement from tune vs. RF-freq: 2.2/4.1 (set:4/3).
- Energy acceptance: Variation of RF voltage and measurement of Lifetime vs. Synchtrotron tune. Difficult due to instabilities for low voltage.
- Measurement of dispersion at pinhole by variation of RF frequency and measurement of beam position.
- Measurements with pinhole and playing with skew quads: all zero gives 47 micron rms beam height, visible rotation of beam and 2.8 hrs lifetime. optimum skew quad setting to suppress rotation (at pinhole location only) gives 42 micron and 3.4 hrs lifetime (for 85mA in 90 buckets F4TJB). Application crashes due to time outs.
- Analysis of lifetime: Measurement while moving slowly in *horizontal* scraper.
- Copying all DBPM software from /work to /prod through the "u.py" backdoor. Reboot of all DBPM crates from /prod.

Returned to F4T nominal optics afterwards, no time to establish F4TJB thoroughly.

FIGURE 5. Reality Check (courtesy of the Swiss Light Source).

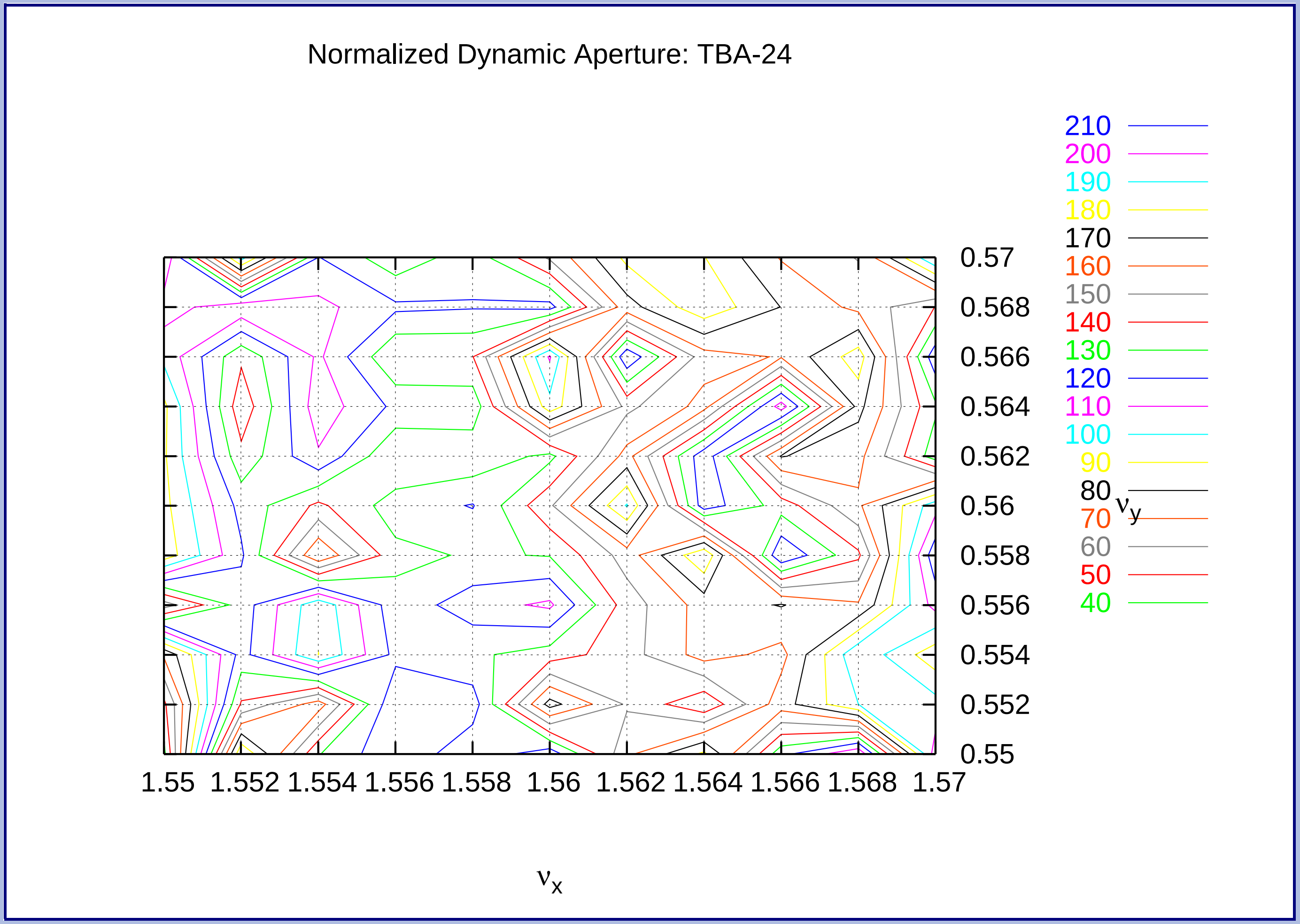

FIGURE 6. Tune Scan for NSLS-II Prototype TBA-24 Cell.

6.0 Closing the Loop: Accelerator Control

6.1 Frequency Map Analysis

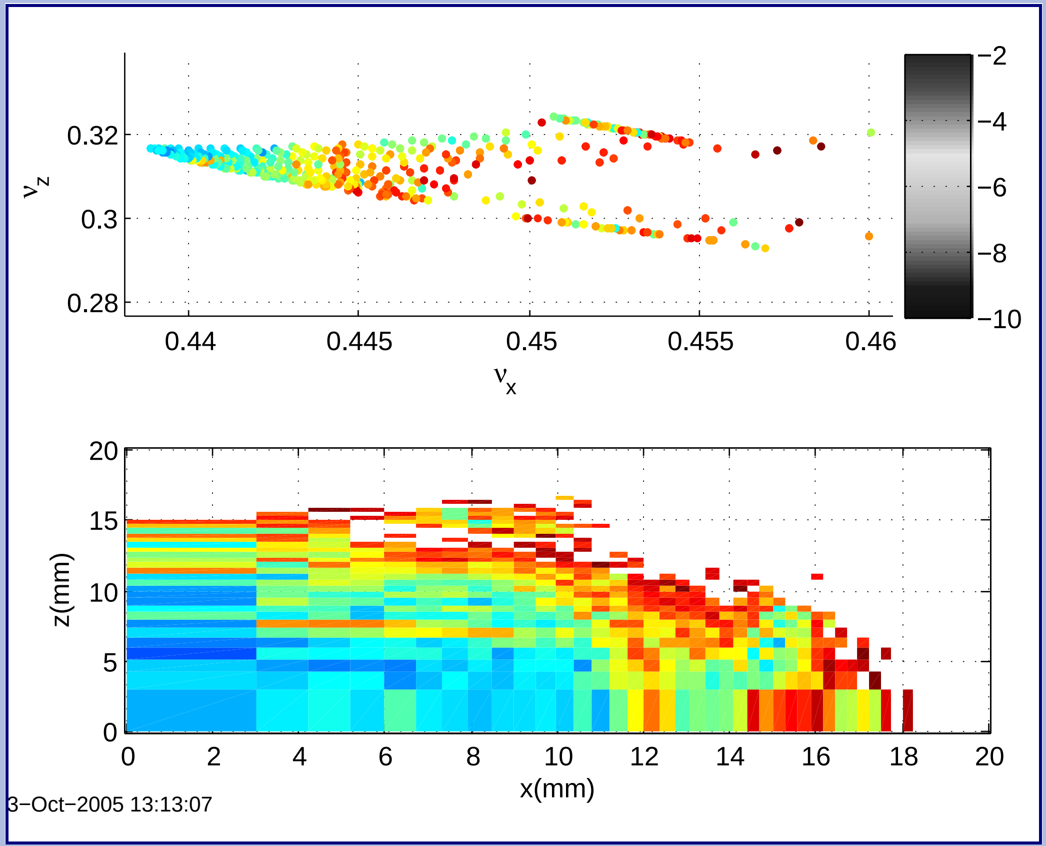

FIGURE 7. Frequency Map Analysis for NSLS-II Prototype TBA-24_6m Cell.

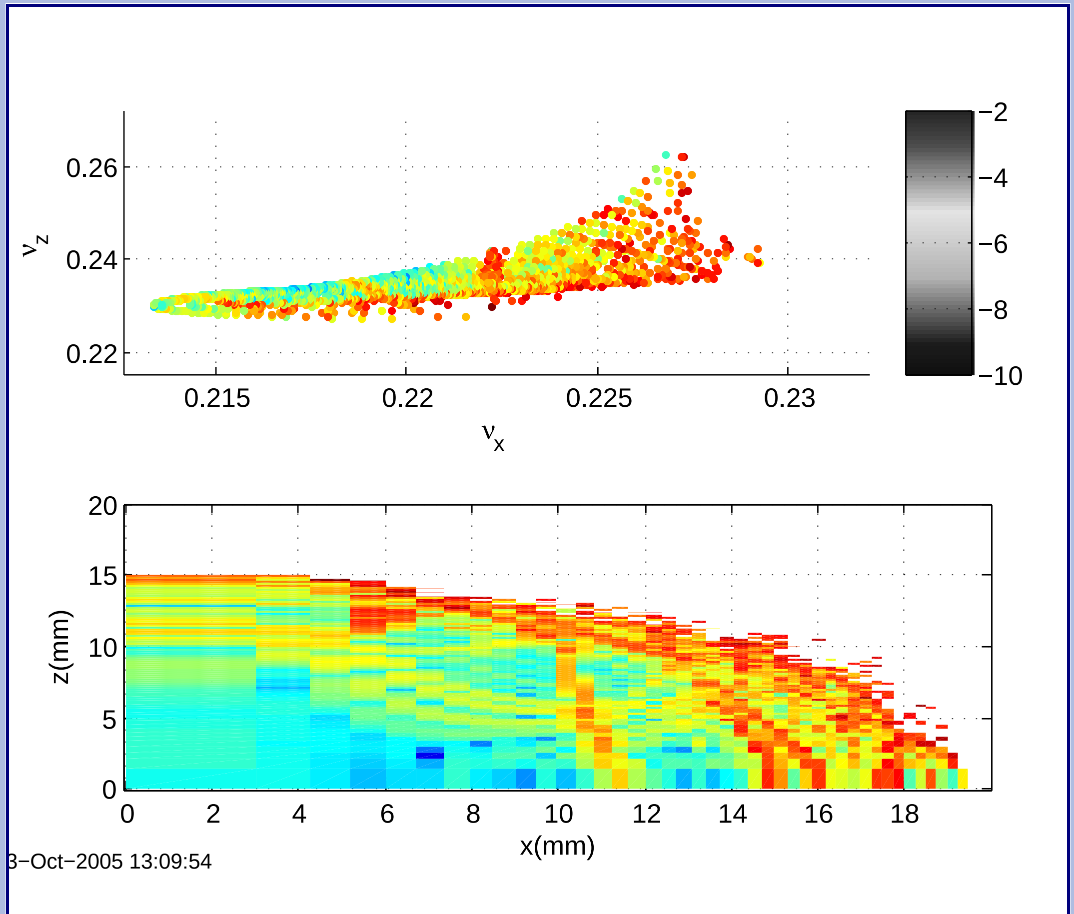

FIGURE 8. Frequency Map Analysis for NSLS-II Prototype DBA-32 Cell.

6.2 Frequency Analysis of Turn-by-Turn BPM Data

The nonlinear dynamics can be observed by displacing the beam with a fast kicker (or at injection) and sampling the center-of-charge motion with a BPM on a turn-by-turn basis. Moreover, with two adjacent BPMs, (x, x') can be estimated and the variation of the linear invariant computed and monitored on a turn-by-turn basis.

The N-turn map

$$\boxed{\mathcal{M}^N = \mathcal{A}^{-1}\left(e^{:h(\vec{J},\vec{\phi}):}\mathcal{R}\right)^N\mathcal{A} \cong \mathcal{A}^{-1}e^{:-g(\vec{J},\vec{\phi}):}e^{:Nk(\vec{J}):}\mathcal{R}\, e^{:g(\vec{J},\vec{\phi}):}\mathcal{A}}$$

leads to

$$J_x(N) = J_x + g_{2100} + g_{1011} + g_{3000} + g_{1002} + g_{1020} + O(b_3^2),$$
$$J_y(N) = J_y - g_{1002} + 2g_{1020} + O(b_3^2)$$

where

$$g_{jklm}(N) = \frac{A_{jklm}(2J_x)^{(j+k)/2}(2J_y)^{(l+m)/2}}{\sin(\pi[(j-k)\nu_x + (l-m)\nu_y])}\cos[\hat{\phi}_{jklm} + (j-k)(\phi_x + N2\pi\nu_x) + (l-m)(\phi_y + N2\pi\nu_y)],$$
$$\hat{\phi}_{ijkl0} = \phi_{ijkl} - \pi[(i-j)\nu_x + (k-l)\nu_y]$$

The Swiss Light Source design is based on:

- direct control of the systematic first order generators by symmetry (second order achromat) and 9 sextupole families to de-couple the cell phase advance, i.e. to free working point,
- and indirect control of the second order by minimizing the cross terms

for a robust dynamic aperture. However, since engineering tolerances perturb the symmetry of the optics, i.e. excite non-structural resonances, the following strategy was outlined as part of the conceptual design.

Three first order modes were deliberately excited:

f	A_x	ϕ_x [deg]	A_y	ϕ_y [deg]
$3\nu_x$	5.0×10^{-9}	-45.0	-	-
$\nu_x - 2\nu_y$	3.0×10^{-9}	90.0	5.0×10^{-9}	-90.0
$\nu_x + 2\nu_y$	1.0×10^{-9}	45.0	2.0×10^{-9}	45.0

Table 5: Deliberate Excitation of First Order Sextupolar Modes.

and simulated by tracking:

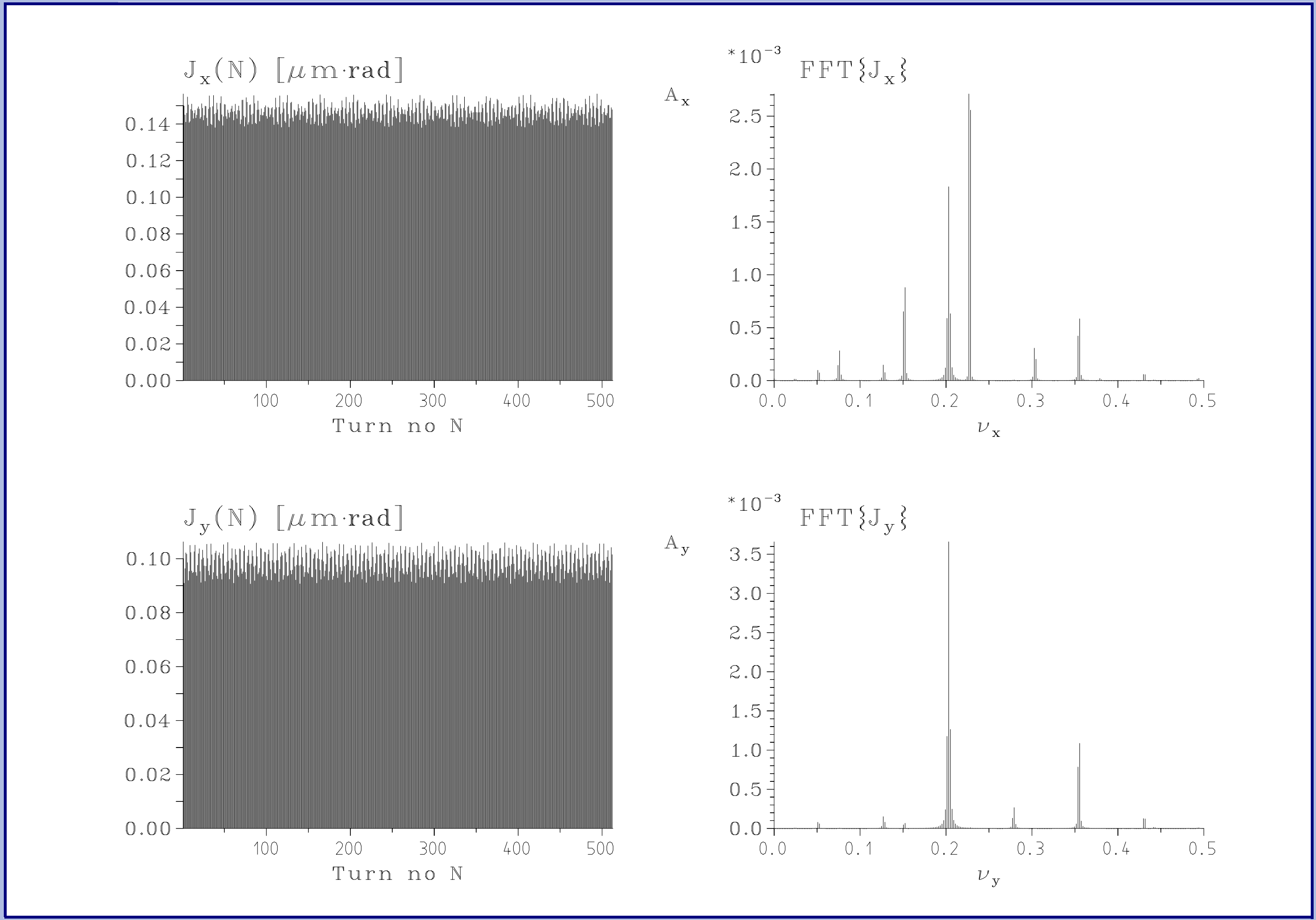

FIGURE 9. Variation of the Linear Invariant (SLS Note 9/97).

FFT (with interpolation, courtesy of E. Asseo, LEAR) of the linear invariant gave:

f	A_x	ϕ_x [deg]	A_y	ϕ_y [deg]
$3\nu_x$	5.3×10^{-9}	-45.1	-	-
$\nu_x - 2\nu_y$	2.9×10^{-9}	-82.6	5.8×10^{-9}	94.6
$\nu_x + 2\nu_y$	1.0×10^{-9}	49.6	1.9×10^{-9}	49.0

Table 6: Estimated First Order Sextupolar Modes from Tracking ($\nu_x - 2\nu_y$ appears with the opposite sign due to aliasing).

Conclusion:

The sextupolar modes for a real accelerator can be measured, and hence compensated, i.e. by introducing a correction with the opposite sign. In particular, if the sextupoles have individual power supplies.

6.3 Beam Response Matrix Measurements

Global matching of the normalized ring (courtesy of G. Portmann's "Middle Layer"): Control of Gradient Errors (horizontal emittance and beam life time):

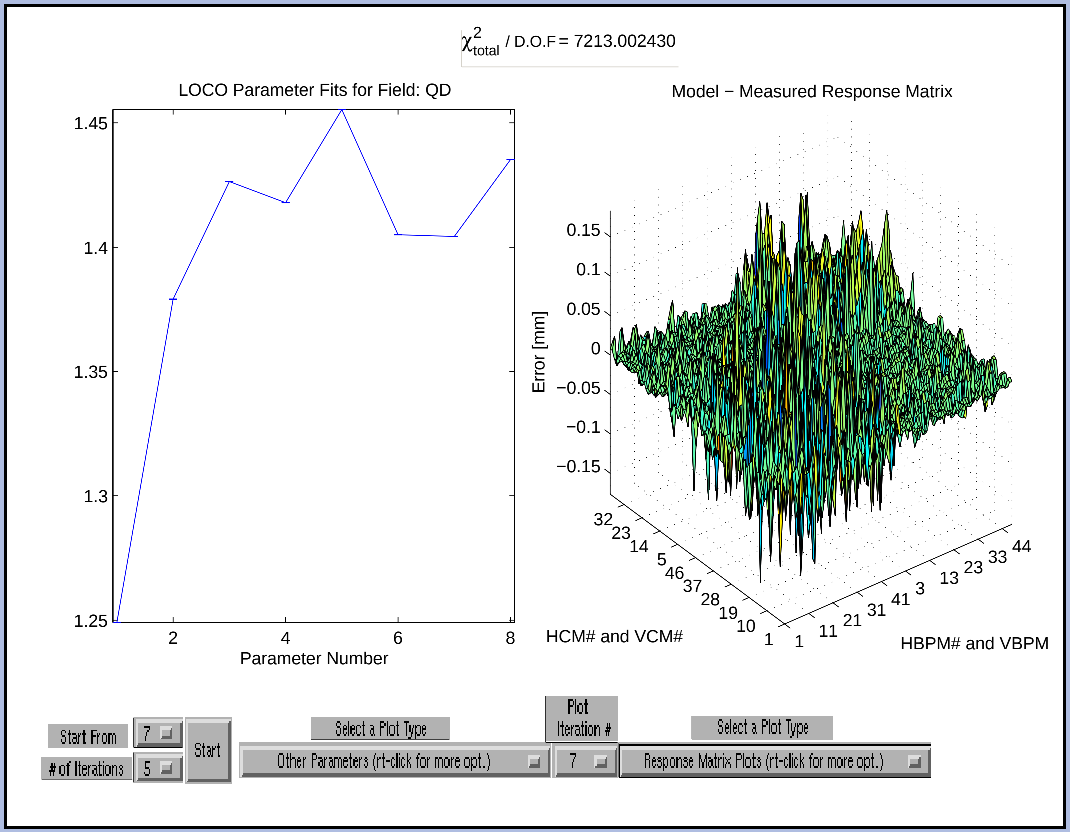

FIGURE 10. Optics Symmetrization of the NSLS X-Ray Ring.

Lattice calibration: control of linear coupling and vertical dispersion (vertical emittance).

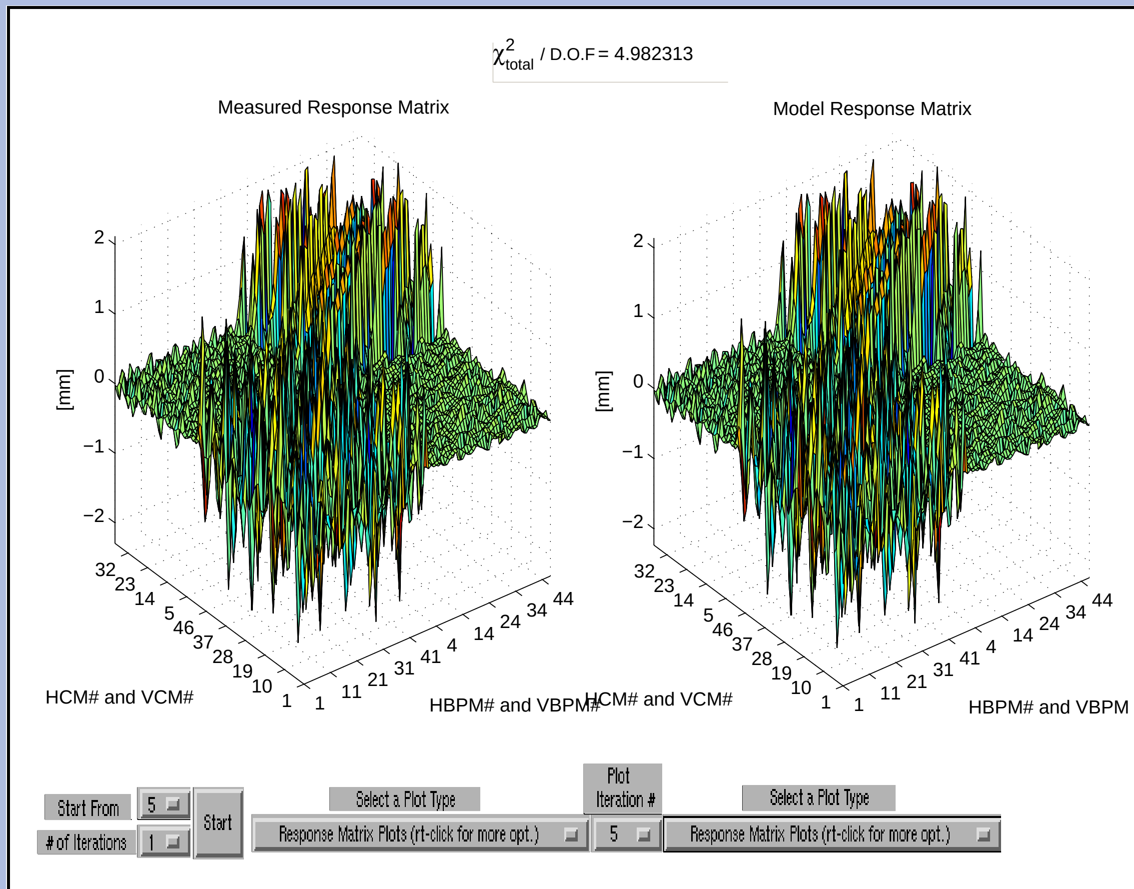

FIGURE 11. Lattice Calibration of the NSLS X-Ray Ring (sextupoles zeroed).

Least-square fit of gain- and roll errors for BPMs and correctors (~500 parameters).

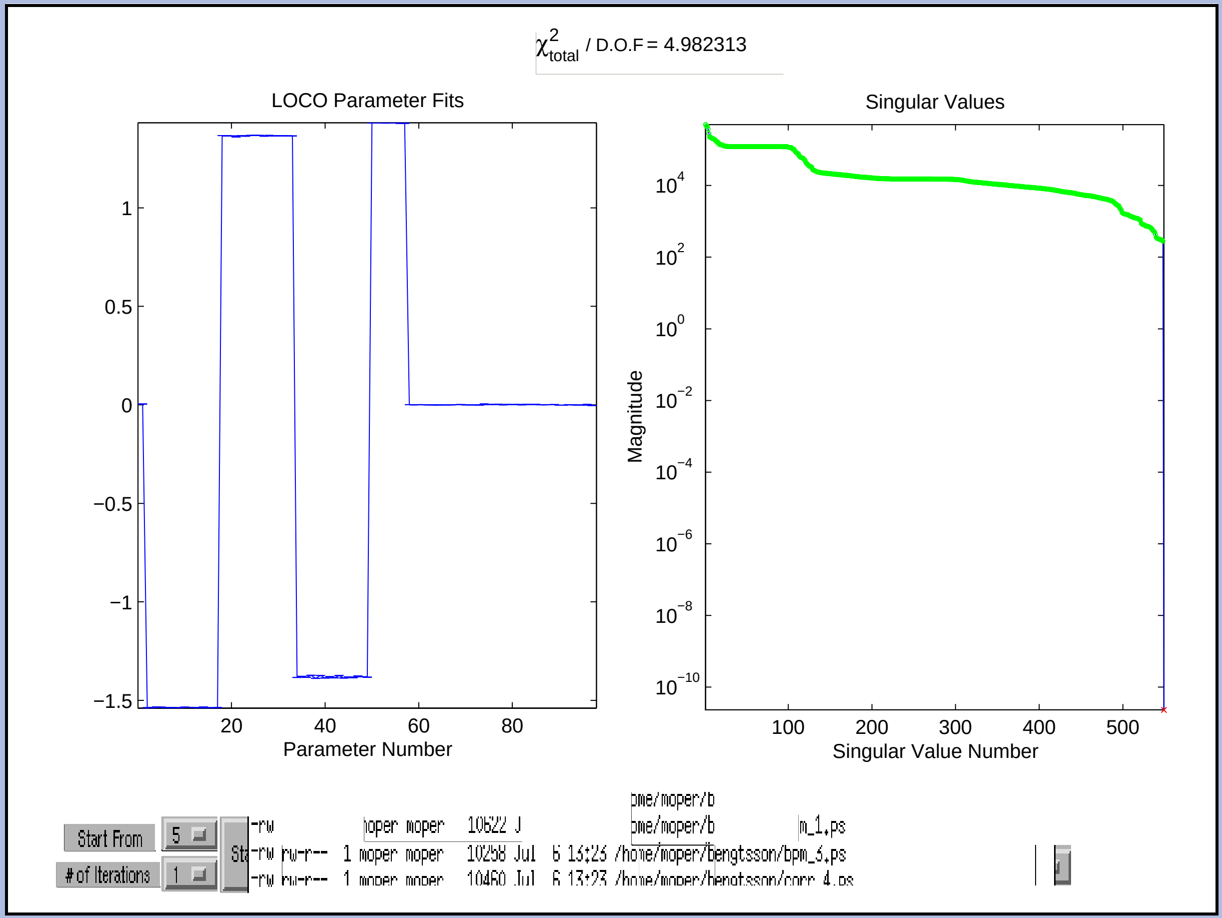

FIGURE 12. Lattice Calibration of the NSLS X-Ray Ring (sextupoles zeroed).

7.0 Conclusions

- While mathematical theorems for stability of nonlinear systems are of limited use for real systems, they are nevertheless useful as a source of inspiration.
- Robust design, i.e. control of a complex dynamical system, requires rigorous studies of its behavior under various conditions, by computer simulations and/or prototyping and testing. In the case of the system of ordinary differential equations for single particle dynamics, a comprehensive model can be implemented.
- Application of Lie series- and map normal form techniques provides an analytical framework to gain insight into the underlying state-space dynamics.
- Application of signal-processing techniques enables one to monitor the state-space dynamics of a real system or simulations, and hence to improve its performance by model driven control; for both design and operations.
- Application of these techniques to the NSLS-II lattice design is starting to bear fruit.